

# Soret and Dufour Effects on Mixed Convective Heat and Mass Transfer Flow in a Rectangular Duct with Heat Generating Sources

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## Abstract

In this paper we investigated the combined influence of dissipation, heat sources, Soret and Dufour effects on the convective heat and mass transfer flow of a viscous fluid through porous medium in a rectangular cavity using Darcy model. Making use of the incompressibility the governing non-linear coupled equations for the momentum, energy and diffusion are derived in terms of the non-dimensional stream function, temperature and concentration. The Galerkin finite element analysis with linear triangular elements is used to obtain the Global stiffness matrices for the values of stream function, temperature and concentration. These coupled matrices are solved using iterative procedure and expressions for the stream function, temperature and concentration are obtained as linear combinations of the shape functions. The behaviour of temperature, concentration, Nusselt number and Sherwood number are discussed computationally for different values of the non-dimensional governing parameters.

**Keywords:** Soret and Dufour effects, heat sources, rectangular duct, finite element analysis.

## 1. Introduction

Natural convection is of great importance in many industrial applications. Convection plays a dominant role in crystal growth in which it affects the fluid-phase composition and temperature at the phase interface that results in a single crystal since poor crystal quality is due to turbulence. It is the foundation in modern electronics industry to produce pure and perfect crystals to make transistors, lasers rods, microwave devices, infrared detectors, memory devices, and integrated circuits. Natural convection adversely affects local growth conditions and enhances the overall transport rate. The combination of temperature and concentration gradients in the fluid will lead to buoyancy-driven flows. This has an importance influence on the solidification process in a binary system. When heat and mass transfer occurs simultaneously, it leads to a complex fluid motion called double-diffusive convection.

Double-diffusion occurs in a wide range of scientific fields such as oceanography, astrophysics, geology, biology and chemical processes. Ostrach (1887) reported complete reviews on the subject. Bejan (1985) reported a fundamental study of scale analysis relative to heat and mass transfer within cavities submitted to horizontal combined and pure temperature and concentration gradients. Kamotani et al. (1985) considered an experimental study of natural convection in shallow enclosures with horizontal temperature and concentration gradients. Other experimental studies dealing with thermo solutal convection in rectangular enclosures were reported by Lee et al. (1990). Hyun et al., (1990) reported numerical solutions for unsteady double-diffusive convection in a rectangular enclosure with aiding and opposing temperature and concentration gradients that were in good agreement with reported experimental results. Other related numerical studies dealing with double-diffusive natural convection in cavities were considered by Beghein et al. (1992) and Nishimura et al. (1998). Bera et al. (1998) studied numerically the heat and mass transfer in an anisotropic porous enclosure due to constant heating and cooling. Morega et al (1996) studied double-diffusive convection by chebyshev collocation method, Technol. Electrically conducting fluids in the presence of a magnetic field have been used extensively in many applications such as crystal growth. Oreper and Szekely (1983) have found that the presence of a magnetic field can suppress natural convection currents and that the factors in determining the quality of the crystal. Alchaar et al. (1995) have considered natural convection heat transfer in a rectangular enclosure with a transverse magnetic field. Rudraiah et al. (1995) and Al-Najem et al. (1998) have studied the effects of a magnetic field on free convection in a rectangular enclosure. Researchers Sandeep and Sugunamma (2013), Mohankrishna et al. (2014), Sugunamma and Sandeep (2011), Jayachandra Babu et al. (2015) considered dissipative and radiating fluids and analyzed the flow and heat transfer behaviour through different channels.

Natural convection heat transfer induced by internal heat generation has recently received considerable attention because of numerous applications in geophysics and energy-related engineering problems. Such applications include heat removal from nuclear fuel debris, underground disposal of radioactive waste materials, storage of foodstuff, and exothermic chemical reactions in packed-bed reactor. Kakac et al. (1985) gave fundamentals and applications of natural convection. Strach (1980) analyzed natural convection with combined driving forces. Churbanov et al. (1994) studied numerically unsteady natural convection of a heat generating fluid in a vertical rectangular enclosure with isothermal or adiabatic rigid walls. Their results were obtained

using a finite-difference scheme in the two-dimensional stream function-velocity formulation. Steady-state as well as oscillating solutions were obtained and compared with other numerical and experimental published data. Other related works dealing with temperature-dependent heat generation effects can be found in the papers by Vajravelu and Nayfeh (1992) and Chamkha (1999). Sandeep et al. (2012), Sulochana and Sandeep (2015) discussed the heat transfer behaviour of some base and nanofluids in presence of radiation and chemical reaction.

Literature suggests that the effect of viscous dissipation on heat transfer as been studied for different geometries. Thermal radiation plays a significant role in the overall surface heat transfer where convective heat transfer is small. Badruddin et al. (2006) have investigated the radiation and viscous dissipation on convective heat transfer in porous cavity. Nishimura et al. (1996) studied the oscillatory double-diffusive convection in a rectangular enclosure with combined horizontal temperature and concentration gradients. Santhi (2011) has investigated double diffusive flow in a rectangular cavity using Darcy model. She has analysed the effect of dissipation radiation on the double diffusive flow of a viscous fluid in the rectangular cavity. Chamka et al. (2002) have investigated the hydromagnetic double-diffusive convection in a rectangular enclosure with opposing temperature and concentration gradients. Sandeep and Sulochana (2015) presented dual solutions for radiative MHD nanofluid flow over an exponentially stretching sheet with heat generation/absorption. Sandeep et al. (2013) studied the radiation effect on the nanofluid flow over a vertical channel. Raju et al. (2015) discussed unsteady boundary layer flow of thermophoretic MHD nanofluid past a stretching sheet with space and time dependent internal heat source/sink. Radiation and solet effects of MHD nanofluid flow over a moving vertical plate in porous medium studied by Raju et al. (2015).

In this paper an attempt has been made to discuss the combined influence of dissipation, heat sources, Soret and Dufour effects on the convective heat and mass transfer flow of a viscous fluid through a porous medium in a rectangular cavity using Darcy model. Making use of the incompressibility the governing non-linear coupled equations for the momentum, energy and diffusion are derived in terms of the non-dimensional stream function, temperature and concentration. The Galerkin finite element analysis with linear triangular elements is used to obtain the Global stiffness matrices for the values of stream function, temperature and concentration. These coupled matrices are solved using iterative procedure and expressions for the stream function, temperature and concentration are obtained as a linear combinations of the shape functions. The behaviour of temperature, concentration, Nusselt number and Sherwood number are discussed.

## 2. Mathematical Formulation

We consider the mixed convective heat and mass transfer flow of a viscous incompressible fluid in a saturated porous medium confined in the rectangular duct (Fig. 1) whose base length is  $a$  and height  $b$ . The heat flux on the base and top walls is maintained constant. The Cartesian coordinate system  $O(x, y)$  is chosen with origin on the central axis of the duct and its base parallel to  $x$ -axis. We assume that the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium. There is no phase change of the fluid in the medium. The properties of the fluid and of the porous medium are homogeneous and isotropic. The porous medium is assumed to be closely packed so that Darcy's momentum law is adequate in the porous medium. The Boussinesq approximation is applicable. Under these assumption the governing equations are given by

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$u' = -\frac{k}{\mu} \left( \frac{\partial p'}{\partial x'} \right) \quad (2)$$

$$v' = -\frac{k}{\mu} \left( \frac{\partial p'}{\partial y'} + \rho' g \right) \quad (3)$$

$$\rho_{\sigma} c_p \left( u' \frac{\partial T'}{\partial x'} + v' \frac{\partial T'}{\partial y'} \right) = K_1 \left( \frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2} \right) + Q(T_0 - T) + \left( \frac{\mu}{K} \right) (u'^2 + v'^2) + k_{12} \left( \frac{\partial^2 C}{\partial x'^2} + \frac{\partial^2 C}{\partial y'^2} \right) \quad (4)$$

$$\rho_{\sigma} c_p \left( u' \frac{\partial C}{\partial x'} + v' \frac{\partial C}{\partial y'} \right) = D_1 \left( \frac{\partial^2 C}{\partial x'^2} + \frac{\partial^2 C}{\partial y'^2} \right) + k_{11} \left( \frac{\partial^2 T}{\partial x'^2} + \frac{\partial^2 T}{\partial y'^2} \right) \quad (5)$$

$$\rho' = \rho_0 \{1 - \beta(T' - T_0) - \beta^*(C' - C_0)\} \quad (6)$$

$$T_0 = \frac{T_h + T_c}{2}, C_0 = \frac{C_h + C_c}{2}$$

where  $u'$  and  $v'$  are Darcy velocities along  $\theta(x, y)$  direction.  $T'$ ,  $C, p'$  and  $g'$  are the temperature, Concentration, pressure and acceleration due to gravity,  $T_c, C_c$  and  $T_h, C_h$  are the temperature and Concentration on the cold and warm side walls respectively.  $\rho', \mu, \nu$ , and  $\beta$  are the density, coefficients of viscosity, kinematic viscosity and thermal expansion of the fluid,  $k$  is the permeability of the porous medium,  $K_1$  is the thermal conductivity,  $C_p$  is the specific heat at constant pressure,  $Q$  is the strength of the heat source,  $k_{11}$  and  $k_{12}$  are the cross diffusivities,  $\beta^*$  is the volume coefficient of expansion with mass fraction concentration.

The boundary conditions are

$$\begin{aligned} u' = v' = 0 & \quad \text{on the boundary of the duct} \\ T' = T_c, C = C_c & \quad \text{on the side wall to the left} \\ T' = T_h, C = C_h & \quad \text{on the side wall to the right} \\ \frac{\partial T'}{\partial y} = 0, \frac{\partial C}{\partial y} = 0 & \quad \text{on the top (y = 0) and bottom} \\ u = v = 0 & \quad \text{walls (y = 0) which are insulated.} \end{aligned} \quad (7)$$

We now introduce the following non-dimensional variables

$$\begin{aligned} x' = ax; \quad y' = by; \quad c = b/a \\ u' = (v/a)u; \quad v' = (v/a)v; \quad p' = (v^2 \rho/a^2)p \\ T' = T_0 + \theta(T_h - T_c); \quad C' = C_0 + \phi(T_h - T_c) \end{aligned} \quad (8)$$

The governing equations in the non-dimensional form are

$$u = -\left(\frac{K}{a^2}\right) \frac{\partial p}{\partial x} \quad (9)$$

$$v = -\frac{k}{a^2} \frac{\partial p}{\partial y} - \frac{kag}{v^2} + \frac{kag\beta(T_h - T_c)\theta}{v^2} + \frac{kag\beta^*(C_h - C_c)\phi}{v^2} \quad (10)$$

$$\text{Pr} \left( u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \alpha \theta + E_c (u^2 + v^2) + Du \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) \quad (11)$$

$$Sc \left( u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) = \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + \frac{ScSo}{N} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (12)$$

In view of the equation of continuity we introduce the stream function  $\psi$  as  $u = \frac{\partial \psi}{\partial y}$ ;  $v = -\frac{\partial \psi}{\partial x}$  (13)

Eliminating  $p$  from the equation (9) and (10) and making use of (11) the equations in terms of  $\psi$  and  $\theta$  are

$$\nabla^4 \psi = -Ra \left( \frac{\partial \theta}{\partial x} + N \frac{\partial \phi}{\partial x} \right) \quad (14)$$

$$\text{Pr} \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \alpha \theta + E_c \left( \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 \right) + Du \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) \quad (15)$$

$$Sc \left( \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} \right) = \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + \frac{ScSo}{N} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (16)$$

where

$$G = \frac{g\beta(T_h - T_c)a^3}{\nu^2} \text{ (Grashof number), } Pr = \mu c_p / K_1 \text{ (Prandtl number),}$$

$$\alpha = Qa^2/K_1(\text{Heat source parameter}), \quad Ra = \frac{\beta g (T_g - T_c) Ka}{\nu^2} \text{ (Rayleigh Number),}$$

$$Sc = \frac{\nu}{D} \text{ (Schmidt Number),} \quad So = \frac{k_{11}\beta^*}{\nu\beta} \text{ (Soret parameter),}$$

$$N = \frac{\beta^* (C_h - C_c)}{\beta (T_h - T_c)} \text{ (Buoyancy ratio),} \quad Ec = \left( \frac{a^4}{\mu K K_1 \Delta T} \right) \text{ (Eckert number),}$$

$$Du = \frac{k_{12}\Delta C}{\rho C_p \Delta T} \text{ (Dufour parameter),}$$

The 4 boundary conditions are

$$\frac{\partial \psi}{\partial x} = 0, \frac{\partial \psi}{\partial y} = 0 \text{ on } x = 0 \text{ \& \;} 1 \quad (17)$$

$$\begin{aligned} \theta &= 1, \varphi = 1 \text{ on } x = 0 \\ \theta &= 0, \varphi = 0 \text{ on } x = 1 \end{aligned} \quad (18)$$

### 3. Finite Element Analysis and Solution of the Problem

The region is divided into a finite number of three node triangular elements, in each of which the element equation is derived using Galerkin weighted residual method. In each element  $f_i$  the approximate solution for an unknown  $f$  in the variational formulation is expressed as a linear combination of shape function.  $(N_k^i)_{k=1,2,3}$ , which are linear polynomials in  $x$  and  $y$ . This approximate solution of the unknown  $f$  coincides with actual values at each node of the element. The variational formulation results in a  $3 \times 3$  matrix equation (stiffness matrix) for the unknown local nodal values of the given element. These stiffness matrices are assembled in terms of global nodal values using inter element continuity and boundary conditions resulting in global matrix equation. In each case there are  $r$  distinct global nodes in the finite element domain and  $f_p$  ( $p = 1, 2, \dots, r$ ) is the

$$\text{global nodal values of any unknown } f \text{ defined over the domain then } f = \sum_{i=1}^8 \sum_{p=1}^r f_p \Phi_p^i,$$

where the first summation denotes summation over  $s$  elements and the second one represents summation over the independent global nodes and

$$\begin{aligned} \Phi_p^i &= N_N^i, \text{ if } p \text{ is one of the local nodes say } k \text{ of the element } e_i \\ &= 0, \text{ otherwise.} \end{aligned}$$

$f_p$ 's are determined from the global matrix equation. Based on these lines we now make a finite element analysis of the given problem governed by (14)-(16) subjected to the conditions (17) – (18).

Let  $\psi^i$ ,  $\theta^i$  and  $\phi^i$  be the approximate values of  $\psi$ ,  $\theta$  and  $\phi$  in an element  $\theta_i$ .

$$\begin{aligned} \psi^i &= N_1^i \psi_1^i + N_2^i \psi_2^i + N_3^i \psi_3^i \\ \theta^i &= N_1^i \theta_1^i + N_2^i \theta_2^i + N_3^i \theta_3^i \\ \phi^i &= N_1^i \phi_1^i + N_2^i \phi_2^i + N_3^i \phi_3^i \end{aligned} \quad (19)$$

Substituting the approximate value  $\psi^i$ ,  $\theta^i$  and  $\phi^i$  for  $\psi$ ,  $\theta$  and  $\phi$  respectively in (13), the error

$$\begin{aligned} E_1^i &= \frac{\partial^2 \theta^i}{\partial x^2} + \frac{\partial^2 \theta^i}{\partial y^2} - P \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \theta^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \theta^i}{\partial y} \right) - \alpha \theta + \\ &\quad + E_c \left[ \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 \right] + Du \left( \frac{\partial^2 \phi^i}{\partial x^2} + \frac{\partial^2 \phi^i}{\partial y^2} \right) \end{aligned} \quad (20)$$

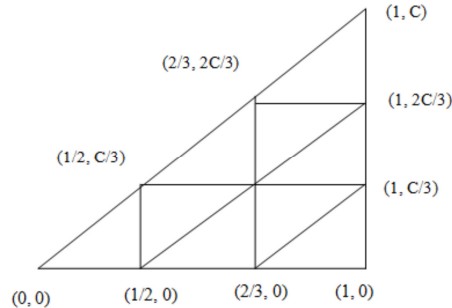
$$E_2^i = \frac{\partial^2 \phi^i}{\partial x^2} + \frac{\partial^2 \phi^i}{\partial y^2} - Sc \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \phi^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \phi^i}{\partial y} \right) + \frac{Sc So}{N} \left( \frac{\partial^2 \phi^i}{\partial x^2} + \frac{\partial^2 \phi^i}{\partial y^2} \right) \quad (21)$$

Under Galerkin method this error is made orthogonal over the domain of  $e_i$  to the respective shape functions (weight functions)

where

$$\int_{e_i} E_1^i N_k^i d\Omega = 0,$$

$$\int_{e_i} E_2^i N_k^i d\Omega = 0$$



$$\int_{e_i} N_k^i \left( \left( 1 + \frac{4}{3N_1} \right) \left( \frac{\partial^z \theta^i}{\partial x^2} + \frac{\partial^z \theta^i}{\partial y^2} \right) - P \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \theta^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \theta^i}{\partial y} \right) - \alpha \theta + \left[ E_c \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 + Du \left( \frac{\partial^z \phi^i}{\partial x^2} + \frac{\partial^z \phi^i}{\partial y^2} \right) \right] \right) d\Omega = 0 \quad (22)$$

$$\int_{e_i} N_k^i \left( \left( \frac{\partial^z \phi^i}{\partial x^2} + \frac{\partial^z \phi^i}{\partial y^2} \right) - Sc \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \phi^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \phi^i}{\partial y} \right) + \frac{ScSo}{N} \left( \frac{\partial^z \theta^i}{\partial x^2} + \frac{\partial^z \theta^i}{\partial y^2} \right) \right) d\Omega = 0 \quad (23)$$

Using Green's theorem we reduce the surface integral (22) & (23) without affecting  $\psi$  terms and obtain

$$\int_{e_i} N_k^i \left\{ \frac{\partial N_k^i}{\partial x} \frac{\partial \theta^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial \theta^i}{\partial y} - P^i N_k^i \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \theta^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \theta^i}{\partial y} \right) - \alpha \theta + E_c \left( \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 \right) + Du \left( \frac{\partial N_k^i}{\partial x} \frac{\partial \phi^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial \phi^i}{\partial y} \right) \right\} d\Omega = \int_{\Gamma_i} N_k^i \left( \frac{\partial \theta^i}{\partial x} n_x + \frac{\partial \theta^i}{\partial y} n_y \right) d\Gamma_i \quad (24)$$

$$\int_{e_i} N_k^i \left\{ \frac{\partial N_k^i}{\partial x} \frac{\partial \phi^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial \phi^i}{\partial y} - Sc^i N_k^i \left( \frac{\partial \psi^i}{\partial y} \frac{\partial \phi^i}{\partial x} - \frac{\partial \psi^i}{\partial x} \frac{\partial \phi^i}{\partial y} \right) + \frac{ScSo}{N} \left( \frac{\partial N_k^i}{\partial x} \frac{\partial \theta^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial \theta^i}{\partial y} \right) \right\} d\Omega = \int_{\Gamma_i} N_k^i \left( \left( \frac{\partial \theta^i}{\partial x} + \frac{ScSo}{N} \frac{\partial \phi^i}{\partial x} \right) n_x + \left( \frac{\partial \theta^i}{\partial y} + \frac{ScSo}{N} \frac{\partial \phi^i}{\partial y} \right) n_y \right) d\Gamma_i \quad (25)$$

where  $\Gamma_i$  is the boundary of  $e_i$ .

Substituting L.H.S. of (19) for  $\psi^i$ ,  $\theta^i$  and  $\phi^i$  in (24) & (25) we get

$$\sum_1 \int_{e_i} \frac{\partial N_k^i}{\partial x} \frac{\partial N_L^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial N_L^i}{\partial y} - P \sum_1 \psi_m^i \int_{e_i} \left( \frac{\partial N_m^i}{\partial y} \frac{\partial N_L^i}{\partial x} - \frac{\partial N_m^i}{\partial x} \frac{\partial N_L^i}{\partial y} \right) d\Omega$$

$$\begin{aligned}
 & -\alpha \sum_{ei} \theta^i \int N_k N_l d\Omega_i + E_c \int_{ei} \left( \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial x} \right)^2 + Du \sum_{\Omega_i} \left( \frac{\partial N_k^i}{\partial x} \frac{\partial \phi^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial \phi^i}{\partial y} \right) \right) d\Omega \\
 & = \int_{\Gamma_i} N_k^i \left( \frac{\partial \theta^i}{\partial x} n_x + \frac{\partial \theta^i}{\partial y} n_y \right) d\Gamma_i = Q_k^i \quad (l, m, k = 1, 2, 3) \quad (26)
 \end{aligned}$$

$$\begin{aligned}
 & \sum_1 \int_{ei} \phi^i \left( \frac{\partial N_k^i}{\partial x} \frac{\partial N_L^i}{\partial x} + \frac{\partial N_L^i}{\partial y} \frac{\partial N_k^i}{\partial y} \right) - Sc \sum_1 \psi_m^i \int_{ei} \left( \frac{\partial N_m^i}{\partial y} \frac{\partial N_L^i}{\partial x} - \frac{\partial N_m^i}{\partial x} \frac{\partial N_L^i}{\partial y} \right) d\Omega \\
 & + \frac{ScSo}{N} \sum_{ei} \theta^i \int \left( \frac{\partial N_k^i}{\partial x} \frac{\partial N_L^i}{\partial x} + \frac{\partial N_L^i}{\partial y} \frac{\partial N_k^i}{\partial y} \right) d\Omega_i \\
 & = \int_{\Gamma_i} N_k^i \left( \frac{\partial \theta^i}{\partial x} + \frac{ScSo}{N} \frac{\partial \phi^i}{\partial x} \right) n_x + \left( \frac{\partial \theta^i}{\partial y} + \frac{ScSo}{N} \frac{\partial \phi^i}{\partial y} \right) n_y d\Gamma_i = Q_i^c \quad (l, m, k = 1, 2, 3) \quad (27)
 \end{aligned}$$

Where

$Q_k^i = Q_{k1}^i + Q_{k2}^i + Q_{k3}^i$ ,  $Q_k^i$ 's being the values of  $Q_k^i$  on the sides  $s = (1, 2, 3)$  of the element  $e_i$ . The sign of  $Q_k^i$ 's depends on the direction of the outward normal w.r.t the element. Choosing different  $N_k^i$ 's as weight functions and following the same procedure we obtain matrix equations for three unknowns ( $Q_p^i$ ) viz.,

$$(a_p^i)(\theta_p^i) = (Q_k^i) \quad (28)$$

where  $(a_{pk}^i)$  is a  $3 \times 3$  matrix,  $(\theta_p^i)$ ,  $(Q_k^i)$  are column matrices.

Repeating the above process with each of  $s$  elements, we obtain sets of such matrix equations. Introducing the global coordinates and global values for  $\theta_p^i$  and making use of inter element continuity and boundary conditions relevant to the problem the above stiffness matrices are assembled to obtain a global matrix equation. This global matrix is  $r \times r$  square matrix if there are  $r$  distinct global nodes in the domain of flow considered.

Similarly substituting  $\psi^i$ ,  $\theta^i$  and  $\phi^i$  in (12) and defining the error

$$E_3^i = \nabla^4 \psi + Ra \left( \frac{\partial \theta}{\partial x} + N \frac{\partial \phi}{\partial x} \right) \quad (29)$$

and following the Galerkin method we obtain

$$\int_{\Omega} E_3^i \psi_j^i d\Omega = 0 \quad (30)$$

Using Green's theorem (26) reduces to

$$\begin{aligned}
 & \int_{\Omega} \left( \frac{\partial N_k^i}{\partial x} \frac{\partial \psi^i}{\partial x} + \frac{\partial N_k^i}{\partial y} \frac{\partial \psi^i}{\partial y} + Ra \left( \theta^i \frac{\partial N_k^i}{\partial x} + \phi^i \frac{\partial N_k^i}{\partial x} \right) \right) d\Omega \\
 & = \int_{\Gamma} N_k^i \left( \frac{\partial \psi^i}{\partial x} n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i + \int_{\Gamma} N_k^i n_x \theta^i d\Gamma_i \quad (31)
 \end{aligned}$$

In obtaining (31) the Green's theorem is applied w.r.t derivatives of  $\psi$  without affecting  $\theta$  terms.

Using (19) and (20) in (31) we have

$$\begin{aligned}
 & \sum_m \psi_m^i \left\{ \int_{\Omega} \left( \frac{\partial N_k^i}{\partial x} \frac{\partial N_m^i}{\partial x} + \frac{\partial N_m^i}{\partial y} \frac{\partial N_k^i}{\partial y} \right) d\Omega + Ra \sum_L \left( \theta_L^i \int_{\Omega} N_k^i \frac{\partial N_L^i}{\partial x} d\Omega + \phi_L^i \int_{\Omega} N_k^i \frac{\partial N_L^i}{\partial x} d\Omega \right) \right\} \\
 & = \int_{\Gamma} N_k^i \left( \frac{\partial \psi^i}{\partial x} n_x + \frac{\partial \psi^i}{\partial y} n_y \right) d\Gamma_i + \int_{\Gamma} N_k^i \theta^i d\Omega_i = \Gamma_k^i \quad (32)
 \end{aligned}$$

In the problem under consideration, for computational purpose, we choose uniform mesh of 10 triangular element (Fig. ii). The domain has vertices whose global coordinates are (0,0), (1,0) and (1,c) in the non-dimensional form. Let  $e_1, e_2, \dots, e_{10}$  be the ten elements and let  $\theta_1, \theta_2, \dots, \theta_{10}$  be the global values of  $\theta$  and  $\psi_1, \psi_2, \dots, \psi_{10}$  be the global values of  $\psi$  at the ten global nodes of the domain (Fig. ii).

#### 4. Results and Discussion

In this analysis we investigate the combined influence of dissipation, heat sources, Soret and Dufour effects on convective heat and mass transfer flow through a porous medium in a rectangular duct. The non-linear coupled equations governing the flow of heat and mass transfer have been solved by using Galerkin Finite Element technique with 3-noded triangular elements. We take the Prandtl number  $Pr = 0.71$  in the analysis. The temperature and concentration have been discussed for different variations of  $Ra, N, \alpha, Sc, So$  and  $Ec$  at different horizontal and vertical levels. The temperature distribution ( $\theta$ ) is shown in figures 1-28 for different parametric values at different horizontal levels  $y = h/3, y = 2h/3$  and vertical levels  $x = 1/2, x = 2/3$ . We follow the convention that non-dimensional temperature  $\theta$  is positive or negative according as the actual temperature  $T$  is  $>$  or  $<$  than  $T_c$ , temperature on the cold wall.

Figs 1-4 represent the temperature  $\theta$  with different values of the Rayleigh number  $Ra$ . It is found that the actual temperature enhances at  $y = h/3$  level and vertical levels  $x = 1/3$  &  $x = 2/3$  and reduces at higher horizontal level  $y = 2h/3$  with increase in  $Ra > 0$ , while increase in  $|Ra| (< 0)$  reduces the actual temperature at  $y = h/3, x = 1/3$  and  $2/3$  levels and enhances at  $y = 2h/3$  level. The effect of heat sources on the temperature  $\theta$  is exhibited in figs 5-8 at different levels. It is found that an increase in the strength of the heat generating sources ( $\alpha > 0$ ) reduces the actual temperature at  $y = h/3, x = 1/3$  &  $x = 2/3$  levels and enhances at  $y = 2h/3$ , while in the case of heat absorbing source the actual temperature enhances at  $y = h/3, x = 1/3$  &  $2/3$  levels and at  $y = 2h/3$  level the actual temperature reduces with  $|\alpha| \leq 4$  and for higher  $|\alpha| \geq 6$  we notice an enhancement in the actual temperature in the vertical strip ( $0.666 \leq 0.798$ ) and reduces in the strip ( $0.846, 0.93$ ).

Figs 9-12 represent  $\theta$  with Soret parameter  $So$ . It is found that the actual temperature reduces with  $So > 0$  at all level, while increase in  $|So| (< 0)$  enhances the actual temperature at  $y = h/3, x = 1/3$  and  $2/3$  levels and reduces at  $y = 2h/3$  level. The influence of Dufour effect ( $Du$ ) on  $\theta$  is shown in figs 13-16 at different levels. It is found that the actual temperature enhances at  $y = h/3, 2h/3$  and  $x = 2/3$  levels and reduces at  $x = 1/3$  level with increase in  $Du$ .

The rate of heat transfer (Nusselt number) on the side  $x = 1$  is shown in the tables 1 and 2 for different parameter variations. It is found that the rate of heat transfer reduces as we move along the vertical direction. The variation of  $Nu$  with Rayleigh number  $Ra$  and  $Sc$  shows that the Nusselt number reduces with increase in  $Ra$  and enhances with  $Sc$  on all the three quadrants. With reference to heat source parameter  $\alpha$  we find that the Nusselt number enhances with increase in the strength of the heat generating source and reduces with that of heat absorbing source at all levels. With reference to buoyancy ratio  $N$ , we find that when the molecular buoyancy force dominates over the thermal buoyancy force, the Nusselt number on the lower and middle quadrants reduces and enhances on the upper most quadrant when the buoyancy forces are in the same direction and for the forces acting in opposite direction the Nusselt number enhances on lower quadrant and reduces on the middle & upper quadrant (table-1). The variation of  $Nu$  with  $Ec$  shows that higher the dissipative heat larger the Nusselt number on the lower and middle quadrants and reduces on the upper quadrant. With reference to  $So$  we find that the Nusselt number enhances with increase in  $So > 0$  and reduces with  $|So| < 0$  on all the three quadrants. Also higher the diffusion thermo effect smaller the Nusselt number on all the three quadrants.



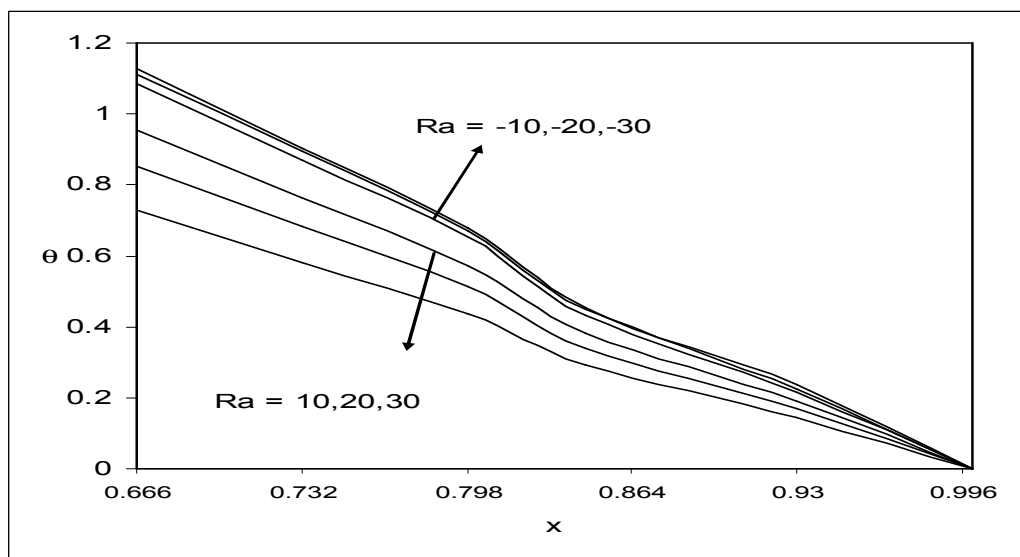


Fig. 1 Variation of  $\theta$  with  $Ra$  at  $y = \frac{2h}{3}$  level  $\alpha=2$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

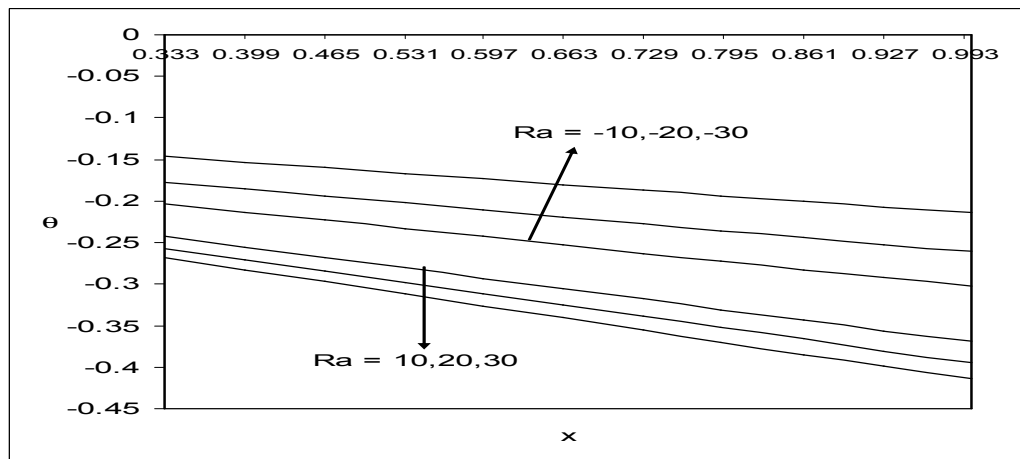


Fig. 2 Variation of  $\theta$  with  $Ra$  at  $y = \frac{h}{3}$  level  $\alpha=2$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

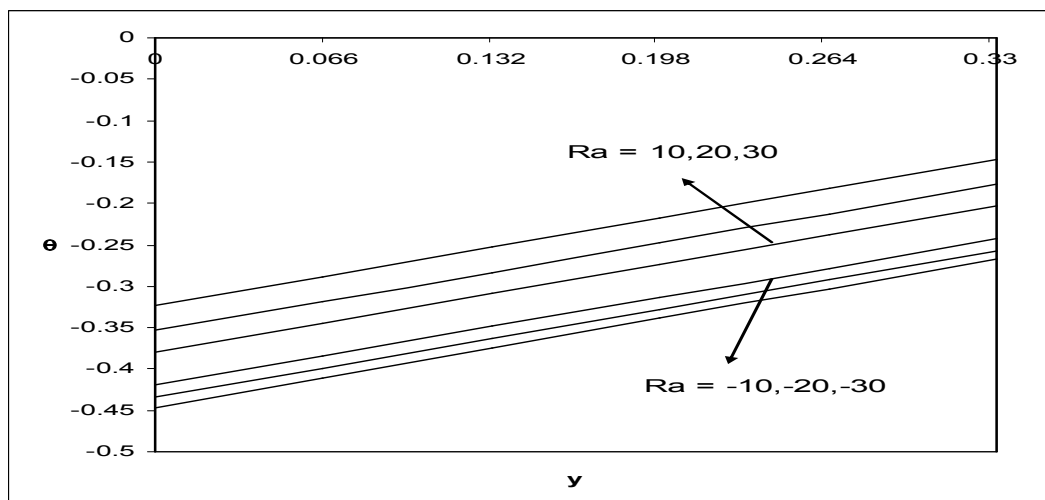


Fig. 3 Variation of  $\theta$  with  $Ra$  at  $x = \frac{1}{3}$  level  $\alpha=2$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$



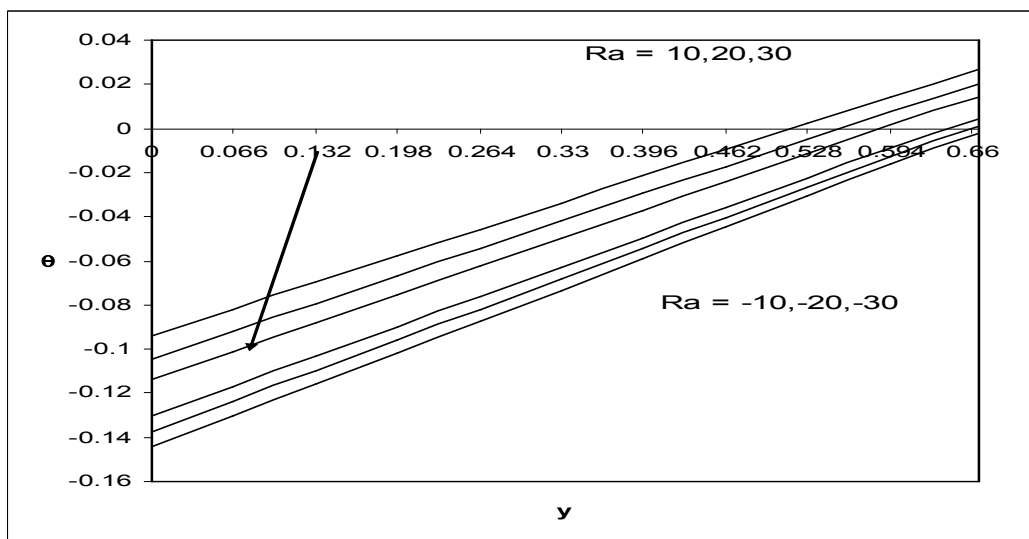


Fig. 4 Variation of  $\theta$  with  $Ra$  at  $x = \frac{2}{3}$  level  $\alpha=2$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

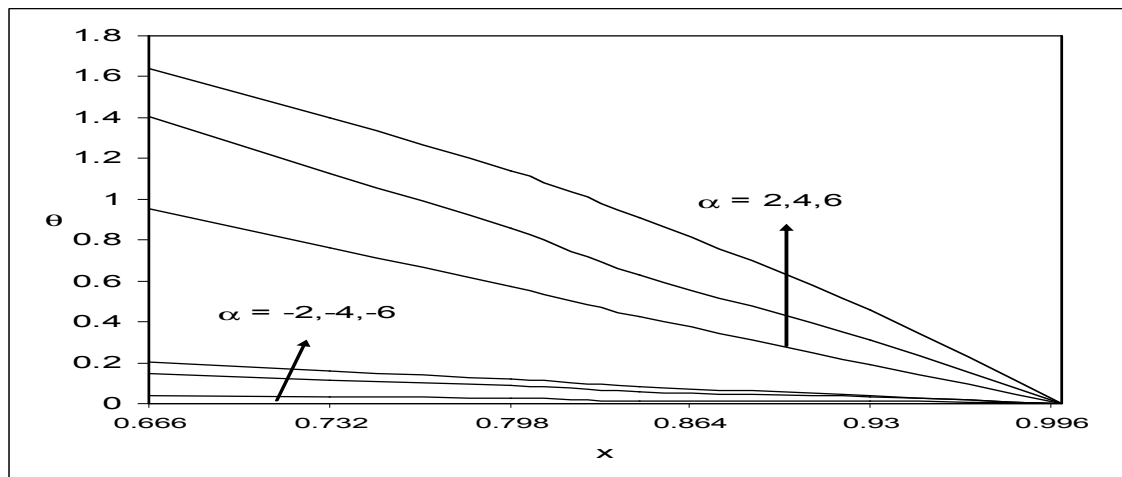


Fig. 5 Variation of  $\theta$  with  $\alpha$  at  $y = \frac{2h}{3}$  level  $Ra=10$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

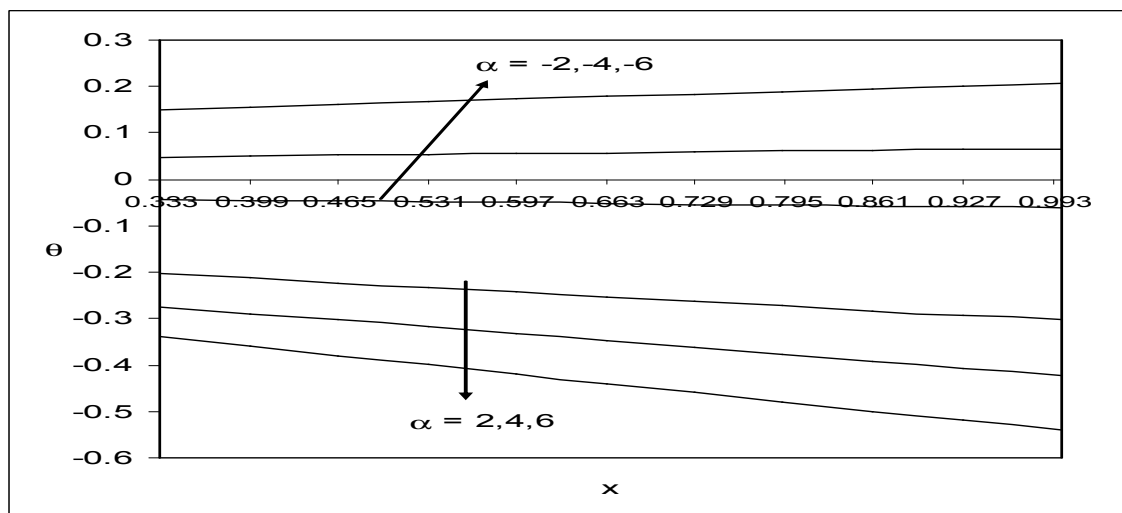


Fig. 6 Variation of  $\theta$  with  $\alpha$  at  $y = \frac{h}{3}$  level  $Ra=10$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

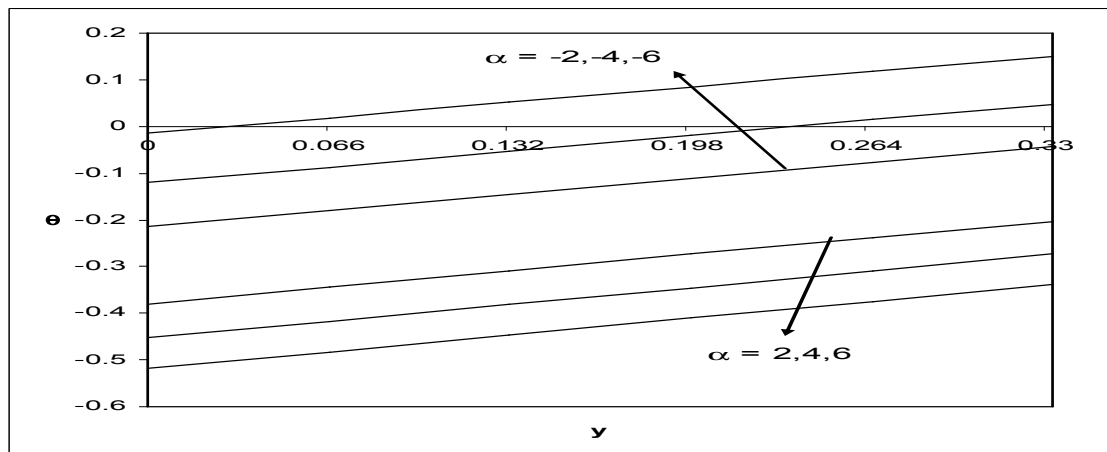


Fig. 7 Variation of  $\theta$  with  $\alpha$  at  $x = \frac{1}{3}$  level  $Ra=10$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

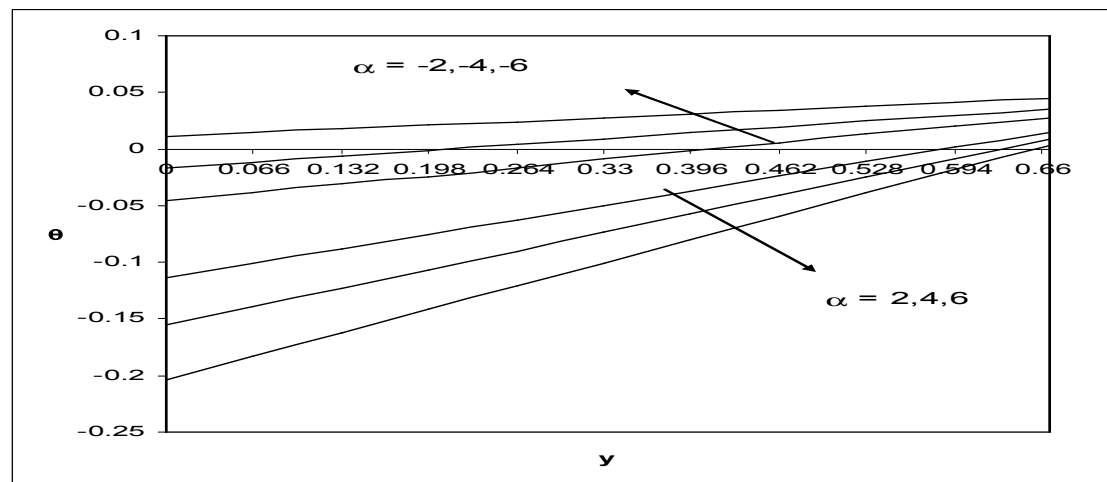


Fig. 8 Variation of  $\theta$  with  $\alpha$  at  $x = \frac{2}{3}$  level  $Ra=10$ ,  $\epsilon=0.1$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

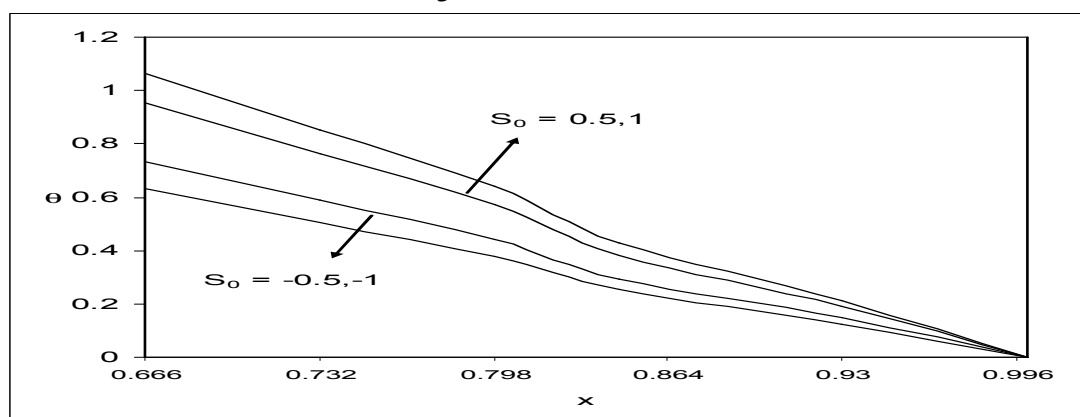


Fig. 9 Variation of  $\theta$  with  $S_0$  at  $y = \frac{2h}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

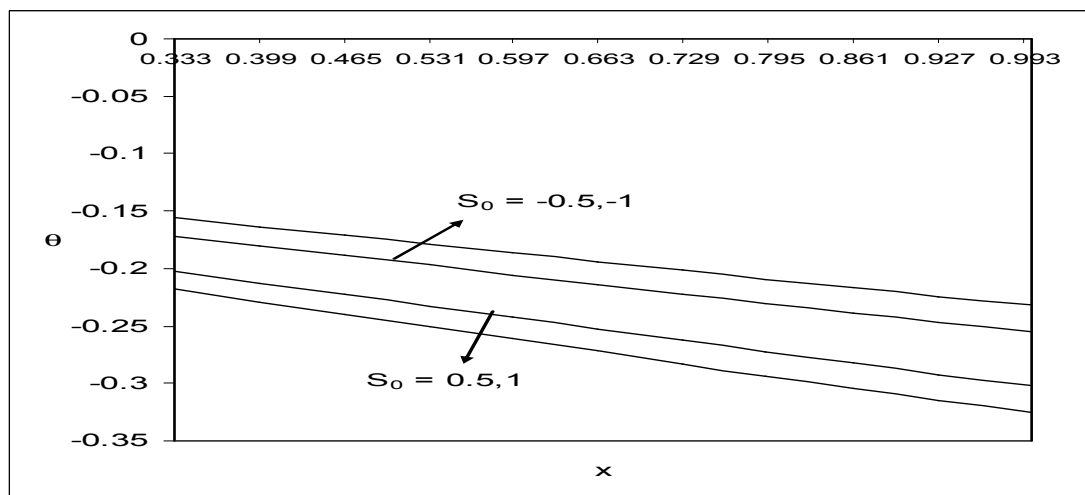


Fig. 10 Variation of  $\theta$  with  $S_0$  at  $y = \frac{h}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

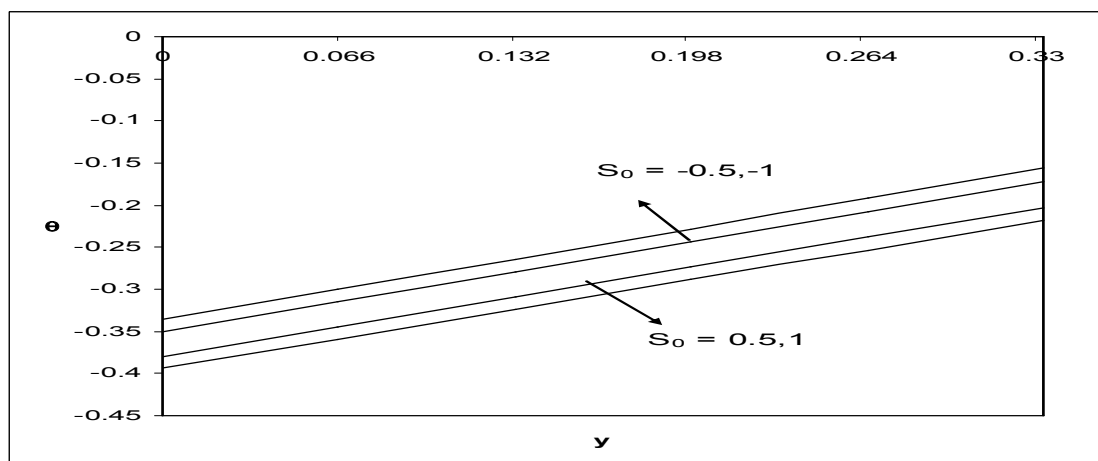


Fig. 11 Variation of  $\theta$  with  $S_0$  at  $x = \frac{1}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

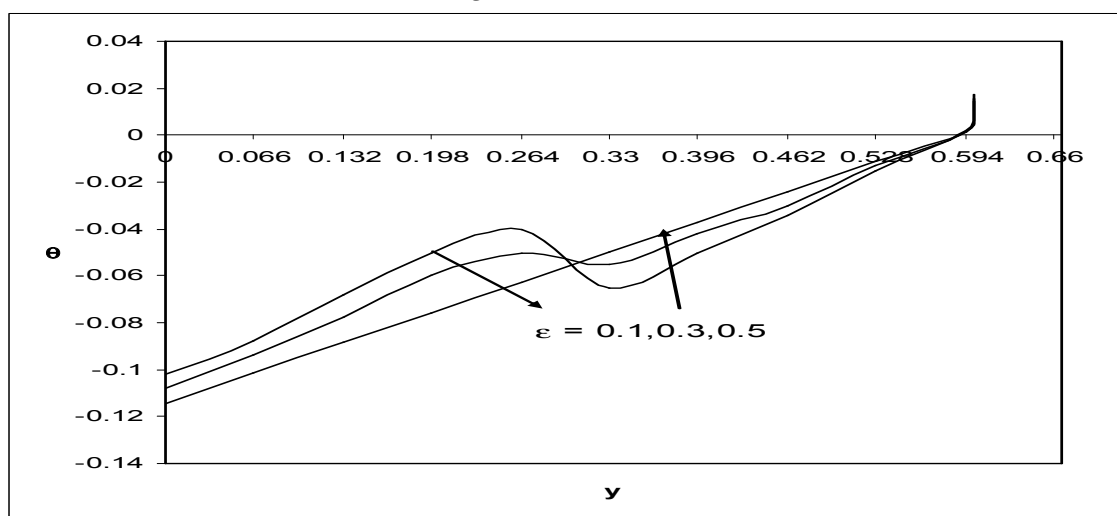


Fig. 12 Variation of  $\theta$  with  $S_0$  at  $x = \frac{2}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $Sc=0.22$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

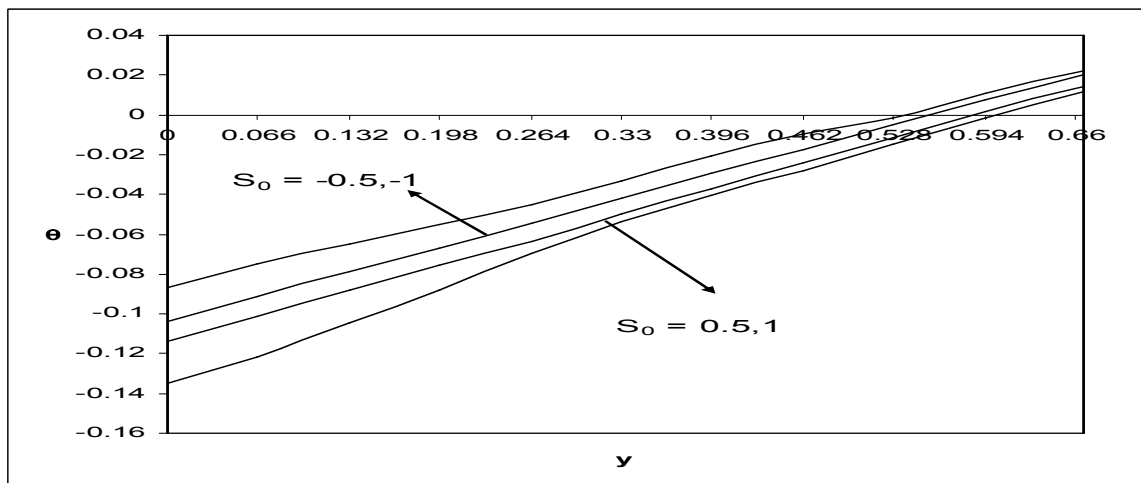


Fig. 13 Variation of  $\theta$  with  $Du$  at  $y = \frac{2h}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $\epsilon=0.1$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

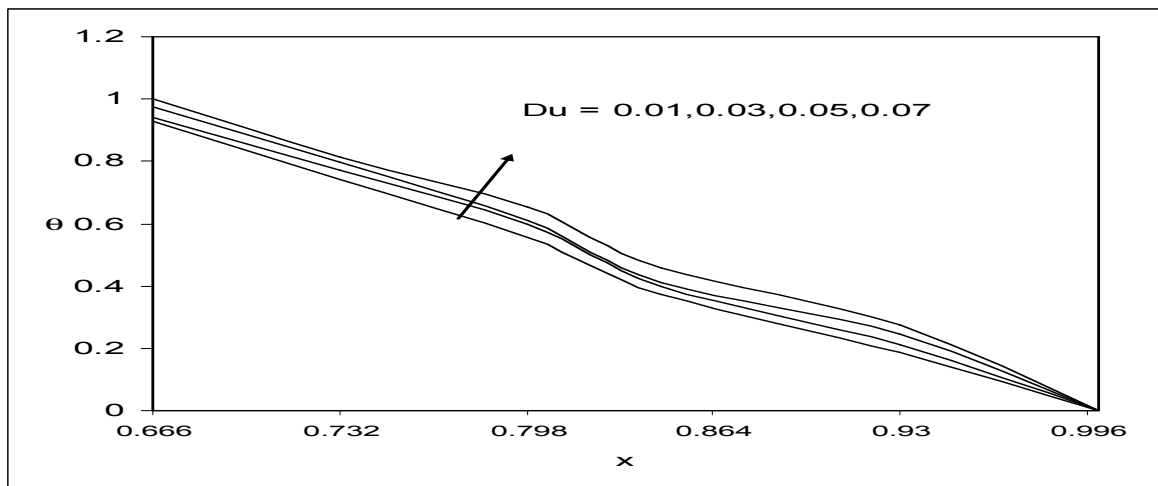


Fig. 14 Variation of  $\theta$  with  $Du$  at  $y = \frac{h}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $\epsilon=0.1$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

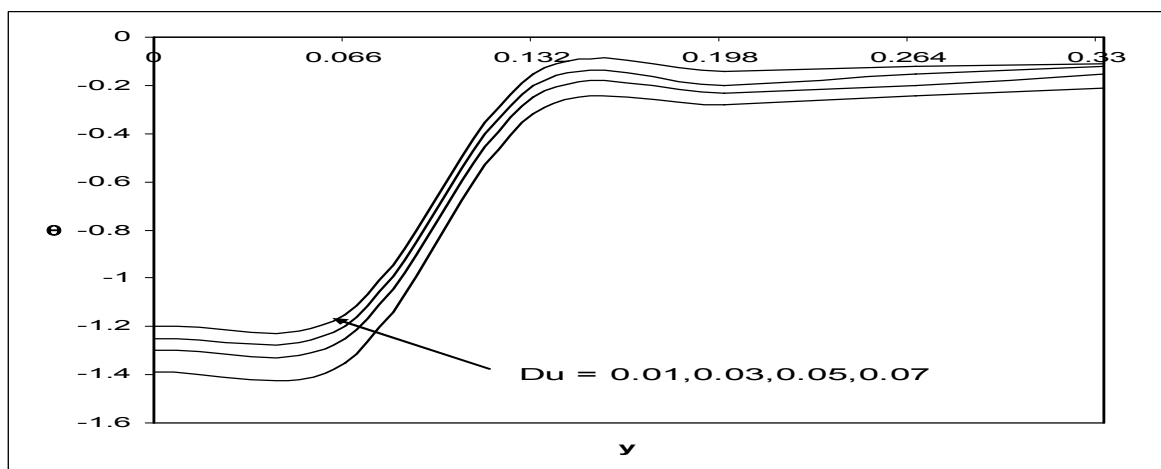


Fig. 15 Variation of  $\theta$  with  $Du$  at  $x = \frac{1}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $\epsilon=0.1$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

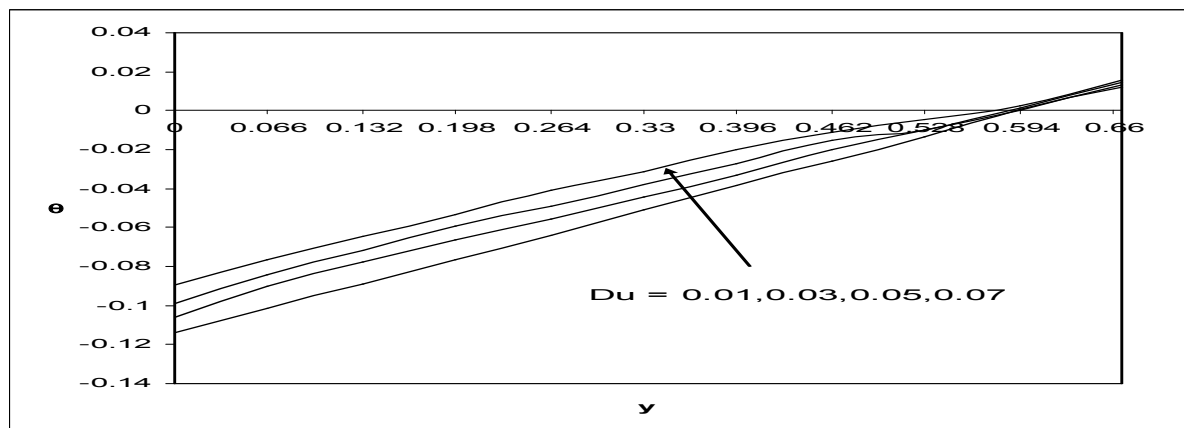


Fig. 16 Variation of  $\theta$  with  $Du$  at  $x = \frac{2}{3}$  level  $Ra=10$ ,  $\alpha=2$ ,  $\epsilon=0.1$ ,  $N=1$ ,  $SB_{0B}=0.5$ ,  $Du=0.01$

|        | I         | II        | III        | IV         | V          | VI        | VII       |
|--------|-----------|-----------|------------|------------|------------|-----------|-----------|
| $Nu_1$ | 3.341757  | 3.313125  | 3.466986   | 3.137157   | 3.050394   | 3.340593  | 3.33255   |
| $Nu_2$ | 3.1493595 | 3.1261704 | 3.2209167  | 3.02723856 | 2.97211446 | 3.1476702 | 3.1468878 |
| $Nu_3$ | 2.9569617 | 2.9392164 | 2.97484725 | 2.9173203  | 2.894694   | 2.9547474 | 2.9612259 |
| $Ra$   | 10        | 20        | 10         | 10         | 10         | 10        | 10        |
| $Sc$   | 1.3       | 1.3       | 1.3        | 1.3        | 1.3        | 0.66      | 1.3       |
| $N$    | 1         | 1         | 1          | 1          | 1          | 1         | 2         |

**Table 2** Nusselt Number  $Nu$  at  $x=2/3$

|        | I         | II        | III       | IV        | V         | VI        |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|
| $Nu_1$ | 3.342975  | 3.345429  | 3.356769  | 3.311673  | 3.2965374 | 3.341676  |
| $Nu_2$ | 3.1498278 | 3.1507659 | 3.1612887 | 3.1257351 | 3.1139961 | 3.1474062 |
| $Nu_3$ | 2.9566818 | 2.9561025 | 2.9658075 | 2.9397981 | 2.932548  | 2.9531352 |
| $Ec$   | 0.01      | 0.03      | 0.01      | 0.01      | 0.01      | 0.01      |
| $So$   | 0.5       | 0.5       | 1         | -0.5      | -1        | 0.5       |
| $Du$   | 0.05      | 0.05      | 0.05      | 0.05      | 0.05      | 0.07      |

## 5. Conclusions

This paper presents the numerical solution of combined influence of dissipation, heat sources, Soret and Dufour effects on the convective heat and mass transfer flow of a viscous fluid through porous medium in a rectangular cavity using Darcy model. Making use of the incompressibility the governing non-linear coupled equations for the momentum, energy and diffusion are derived in terms of the non-dimensional stream function, temperature and concentration. The conclusions of the present study are made as follows:

- An increase in Rayleigh number causes to increase in the temperature profiles in vertical level and reduces in horizontal level.
- A raise in the value of heat source parameter enhances the temperature profiles of the flow.
- For higher values of the dissipative heat enhances the actual temperature.
- Decrease in molecular diffusivity depreciates actual temperature at both the vertical and horizontal levels.

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